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Integrals and derivatives of fractional order and some of their applications. (Integraly i proizvodnye drobnogo poryadka i nekotorye ikh prilozheniya). (Russian)

Minsk: "Nauka i Tekhnika". 688 p. R. 7.60 (1987).

The purpose of the present monograph is to present the whole body of both the classical and modern theory of integrals and derivatives of fractional order, as well as its applications to differential and integral equations on nearly 700 pages.

The book consists of a brief historical account and 8 chapters. In the first five chapters the theory of fractional integration is dealt with, in Chapters 1-4 for functions of one variable, in Chapter 5 for functions of several variables. After introducing the necessary preliminaries concerning integrals and derivatives of fractional (and even complex) order over a real interval, some simple acting problems in Lebesgue und Hölder spaces (with or without weight functions) are discussed in the first chapter. The second chapter is dedicated to fractional integrals and derivatives over the half-axis and the whole real line. Moreover, the acting problem for some classical integral transforms (Fourier, Laplace, Mellin) in spaces of both ordinary and generalized functions is discussed. At the beginning of the third chapter, compositions of fractional integrals with power-exponential weight are considered, as well as relations with singular integrals. Further topics are fractional integrals of potential type, acting problems in Lipschitz classes, fractional derivatives of absolutely continuous functions, and relations with summable series. At the end of Chapter 3, the Leibniz rule and asymptotic expansions of fractional integrals in the case of power- logarithmic and power-exponential asymptotics are discussed. Chapter 4 deals with some generalizations and modifications, such as Erdélyi- Kober operators, fractional integro-differentiation in Hadamard's sense, fractional integrals in Chen's or Dzhrbashjan's sense. Moreover, fractional Weyl integrals and derivatives of periodic functions are discussed, as well as fractional integrals with power-logarithmic kernel functions and those over complex domains. The corresponding "higher dimensional" theory is developed in Chapter 5, including fractional integro-differentiation in M. Riesz' sense, and fractional powers of hyperbolic and parabolic differential operators.

The last three chapters deal with applications to differential equations and integral equations of the first kind. In Chapter 6, generalized Abel equations with power-type kernel functions over the whole axis or an interval are solved, as well as with power-logarithmic kernel functions and variable integration limits (including the Fredholm theory of integral equations of the first kind). Chapter 7 is concerned with integral equations of the first kind involving special kernel functions (both homogeneous and non-homogeneous). Finally, representation theorems for solutions of partial differential equations in terms of analytic functions through fractional integrals are dealt with in Chapter 8, including applications to Euler-Poisson-Darboux equations. At the end of Chapter 8, boundary value problems (of Cauchy and Dirichlet type) for ordinary differential equations of fractional order are studied.

It is not exaggerated to say that this work represents in book form, probably for the first time in the history, all kinds of fractional derivatives and integrals which are presently known in mathematics and engineering sciences. It provides not only a profound comparison of such derivatives and integrals, but also shows that they coincide in many cases which at first glance seem to be quite independent. The authors have made the attempt to collect here practically all published papers in the field; as a result, they present a bibliography of 1421 items covering the period from 1695 to 1987. Moreover, an interesting historical appendix, an authors index, a subject index, an index of symbols, and a detailed table of contents make the book really tractable. To sum up, this work has an exhaustive self-contained encyclopedic character, and will certainly become a standard reference. Hopefully, a rapid translation into English will make it accessible to a wide mathematical audience also in western countries.

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Keywords:

integrals and derivatives of fractional order; integral transforms; Erdélyi-Kober operators; fractional integro-differentiation in Hadamard's sense; fractional Weyl integrals; fractional integro- differentiation in M. Riesz' sense; applications to differential equations and integral equations; Euler-Poisson-Darboux equations

Classification:

∗26A33 Fractional derivatives and integrals (real functions)
44A10 Laplace transform
35Q05 Euler-Poisson-Darboux equation and generalizations
26-02 Research monographs (real functions)
34B99 Boundary value problems for ODE
44A15 Special transforms
45E10 Integral equations of the convolution type
42A38 Fourier type transforms, one variable
45B05 Fredholm integral equations

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