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★**Интегралы и производные дробного порядка и некоторые их приложения. (Russian) [Integrals and derivatives of fractional order and some of their applications]**

Edited and with a preface by S. M. Nikolʹskiĭ.
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In our time, it is very difficult to imagine a wide scientific field which has escaped being covered by a score of lengthy monographs, especially when it concerns mathematics. However, it surprisingly happens that until now such a field, classical in its origin and highly developed, really existed, and the book under review is the first one to cover the gap. The very manner in which it is written seems to be exactly adequate to an extremely difficult task of exposition of the theory of fractional calculus and its applications throughout its more than one-and-a-half century history, starting from the works of its founders—N. H. Abel, J. Liouville, B. Riemann, A. V. Letnikov, H. Weyl, J. Hadamard — and finishing with all the modern results on the subject, to which the authors of the monograph are known contributors. Indeed, besides the brief historical outline preceding the introduction (see the contents below), each chapter of the book contains a section with diverse historical comments and detailed indications to many related results not included, for some reason, in the main body of the chapter. The book is furnished with useful subject and author indexes, and with lists of the main notations and definitions. As for the style, the authors do not economize on demonstrations and calculations, but provide a sufficient number of striking examples wherever needed and manage to combine the high level of generality with the detailed consideration of essential particular cases. In effect, the text achieves almost classical clarity and is easy to read.

The main part of the book contains eight chapters. The first two of them deal with the various definitions of fractional integrals and derivatives of functions of a single variable and with their basic properties. Further results on the fractional calculus in one variable, such as asymptotic expansions of fractional integrals, are considered in Chapter 3. In Chapter 4 other forms of fractional integrals and derivatives are considered. The vast Chapter 5 represents the modern theory of fractional calculus of functions in several variables. In Chapters 6–8 the applications of fractional calculus to both integral and ordinary and partial differential equations are described; unlike the majority of contemporary studies of the subject, the best part of these chapters presents the explicit solutions of the considered equations obtained by means of fractional integrals and derivatives.

Finally, the reviewer dares to say that this book may be a major event in the modern mathematical literature and hopes that, like himself, many people will be able to get a lot of pleasure and profit out of reading it; to provide this possibility, an English translation in the near future is strongly recommended.

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