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★**Equations with involutive operators. (English. English summary)**

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The topic of the book is abstract and concrete equations that may be written in the form $\sum_{j=0}^{n-1} Q^j A_j f = g$, where Q is an involution satisfying $Q^n = I$ and the coefficients A_j belong to classes of operators with well-known Fredholm properties. The authors also embark on equations of the more general form $\sum_{j=0}^{n-1}\sum_{k=0}^{m-1} Q^j P^k A_{jk} f = g$, where Q and P are involutions with $Q^n = I$ and $P^m = I$ that are connected by an equality of the form $QPQ^{-1} = \gamma P^\nu$ with some $\gamma \in \mathbf{C}$. Operators of this type have been extensively studied for many decades. The pioneer work was done by Khalilov, Cherskii, Litvinchuk, Gohberg, Krupnik, and Przeworska-Rolewicz, to mention only a few principal figures. In the first stage of the development, the typical concrete model was the case where Q is the Cauchy singular integral operator S (satisfying $S^2 = I$ on closed curves) and the A_j's are multiplication operators. Subsequent research was significantly inspired by the concrete situation in which Q is a Carleman shift and the A_j's are themselves singular integral operators. Recent investigations cover a very broad spectrum of concrete involutions and coefficient operators, and the book under review is a successful attempt to present the state of the art in a systematic and readable manner.

The authors develop two axiomatic approaches to the Fredholm theory of operators with an involution (including index formulas and, whenever available, invertibility criteria). Thus, the treatment of concrete equations is reduced to the verification of the axioms. Nevertheless this is in general a nontrivial task, which leads to difficult questions and fascinating mathematics.

The concrete one-dimensional equations tackled in the book include singular integral equations with a finite group of shifts, convolution equations with reflection and complex conjugation, discrete convolution equations with oscillating coefficients, and equations with kernels homogeneous of degree -1 and with inversion and complex conjugation. In the higher-dimensional case, the authors consider convolution equations with linear shifts as well as equations with homogeneous kernels and certain shifts. An entire chapter is devoted to singular integral equations on the real line with a fractional linear shift. This is an especially exciting matter, because such shifts generate unbounded operators and hence the treatment of the equations requires a clever strategy and very fine tools.

Many results of the book are due to the authors themselves. The authors' notes on the history of the topic are very informative. The bibliography includes 300 items. A missing item is S. Roch and B. Silbermann's report [Algebras of convolution operators and their image in the Calkin algebra, Akad. Wiss. DDR, Berlin, 1990; MR 92d:47067], which is a fairly popular reference for algebras of operators with Carleman shifts and complex conjugation and for algebras of Wiener-Hopf, Mellin, and Hankel operators.

Overall, this book is a nice demonstration of the usefulness of a unifying abstract theory for a babel of concrete problems on the one hand, and the way an abstract scheme may come to full blossom when illustrated by a wealth of nontrivial concrete realizations on the other. It is a good text for beginners, a valuable source for those who need quick advice in connection with a concrete equation, and it will definitely attract researchers in the field. *A. Böttcher* (D-TUCHM)